

NAG Toolbox for MATLAB

f07gu

1 Purpose

f07gu estimates the condition number of a complex Hermitian positive-definite matrix A , where A has been factorized by f07gr, using packed storage.

2 Syntax

```
[rcond, info] = f07gu(uplo, n, ap, anorm)
```

3 Description

f07gu estimates the condition number (in the 1-norm) of a complex Hermitian positive-definite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – **string**

Indicates how A has been factorized.

uplo = 'U'

$A = U^H U$, where U is upper triangular.

uplo = 'L'

$A = L L^H$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – **int32 scalar**

n , the order of the matrix A .

Constraint: $n \geq 0$.

3: **ap**(*) – **complex array**

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The Cholesky factor of A stored in packed form, as returned by f07gr.

4: **anorm** – double scalar

The 1-norm of the **original** matrix A , which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07gr or else from a copy of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

work, rwork

5.4 Output Parameters

1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **anorm**, 5: **rcond**, 6: **work**, 7: **rwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07gu involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to f07gs with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is f07gg.

9 Example

```
uplo = 'L';
n = int32(4);
ap = [complex(1.797220075561143, +0);
```

```
complex(0.8401864749527325, +1.068316577423342);  
complex(1.057188279741849, -0.467388502622712);  
complex(0.233694251311356, -1.391037210186643);  
complex(1.316353439509685, +0);  
complex(-0.4701749470106329, +0.3130658155999466);  
complex(0.08335250923944192, +0.03676071443037458);  
complex(1.560392977137124, +0);  
complex(0.9359617337923402, +0.9899692192815736);  
complex(0.6603332973655893, +0)];  
anorm = 10.96735730690591;  
[rcond, info] = f07gu(uplo, n, ap, anorm)  
  
rcond =  
    0.0066  
info =  
    0
```
